Abbe Theory of Imaging and Spatial Filtering

Introduction

The experiment on interference and diffraction demonstrated that using ray optics is not sufficient to describe the physical behavior of light waves. Interference and diffraction play a part in the formation of an image. What we will demonstrate in this lab is that it is possible to intervene in the path of the light in such a way as to change the image formed in special ways.

Consider a diffraction grating consisting of equally spaced narrow absorbing and transmitting (black and white) bands or slits. It is possible not only to determine mathematically the directions θ_m of the diffracted orders using:

$$\sin\theta_m = m\lambda/d \tag{1}$$

(with m as an integer), but one can also determine the relative irradiances of the diffracted spots. If we insert a lens behind the diffraction grating, we can redirect the orders of the diffraction grating from infinity to the back focal plane of the lens. We will see how this can be used to understand imaging.

Most of us are familiar with repetitions in time, such as periodic oscillations which produce sound. This oscillation is the source of tones that we hear, and pure tones are made of sine waves of a single frequency. More complicated waveforms can be constructed from the superposition of pure sine waves of different frequencies. In the Fourier representation of this situation, each sine wave has a different location and amplitude in the Fourier spectrum.

The same concept can be applied to spatial frequencies (which are usually measured in cycles/mm or mm^{-1}). The spatial frequency represents the rapidity with which a feature varies spatially across an object or image. The idea is that you can have rapid variations (which represent high spatial frequencies) as well as slow changes or fluctuations which occur on larger spatial scales (these represent low spatial frequencies).

The diffraction grating mentioned above provides a simple example of an object which produces only a few spatial frequencies. If the grating consisted of a sinusoidal variation (see Figure 1a), there would only be a zeroth order and the first orders ($m = \pm 1$). As the repetitive patterns in the grating depart from sinusoidal, additional diffraction orders appear and, in the idealized case of the black and white grating (with alternating perfectly transmitting and absorbing slits), a whole series of diffraction orders is present (Figure 1b).

All of this can be expressed mathematically in terms of Fourier theory. Any periodic (repeating) function can be expressed as a superposition of sine waves at the fundamental frequency (f) and its higher harmonics at integer multiples of f (i.e., $2f$, $3f$, etc.) The amount that each frequency contributes is different and can be calculated using integrals. The decomposition of the periodic function into its harmonic components is called Fourier analysis, and from this analysis the amplitude of each harmonic contribution is determined as is its phase relative to the fundamental (i.e., whether the harmonic is in phase or 180 degrees out of phase with the fundamental).

This procedure can be reversed. If a pattern at the fundamental frequency is combined with the appropriate number of higher harmonics, it is possible to approximate any function with a repetition

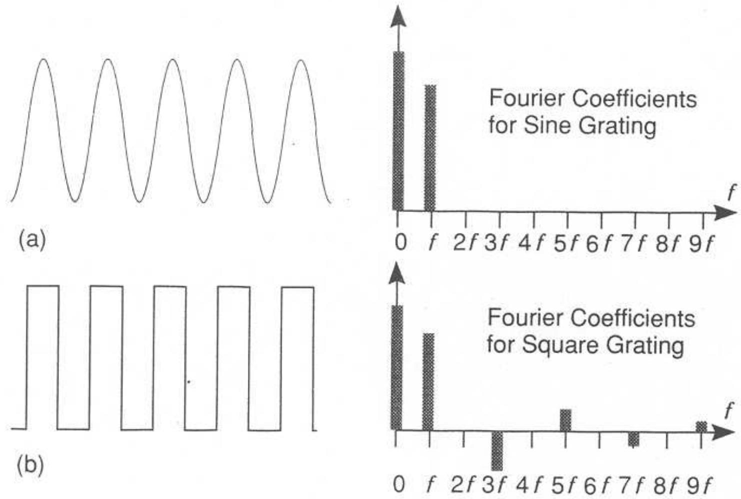

Figure 1: *Sinusoidal grating vs. black and white grating.*

frequency equal to that of the fundamental. This is called Fourier synthesis. To completely synthesize a function such as our idealized alternating black and white grating, an infinite number of harmonics would be needed. If only a finite number of harmonics are used, the synthesized function will resemble the function, but it will have edges that are not as sharp as the original (and it will have ringing oscillations near the sharp edges). A simple example is shown in Figure 2 in which only the fundamental and two harmonics are used to approximate the square wave function, which is equivalent to our black and white grating function. In this lab you will investigate optical techniques that use Fourier analysis and synthesis in creating images.

If a lens is placed behind the diffraction grating which is illuminated by a laser, the diffraction orders can be focused to spots at the focal plane of the lens. The focused spots have intensities (irradiances) that are proportional to the square of the Fourier amplitudes that we could calculate for this diffraction grating. Even if the diffracting object is not a grating or series of lines with repetitive spacings, the light pattern at the focal plane of the lens still describes the content of spatial frequencies found in the diffracting object. Objects that are large and smoothly varying in their shading will not diffract the beam much, and their contributions will lie close to the optical axis of the lens. Objects which are small and have fine detail and sharp edges will cause substantial diffraction, and their contributions will be further from the optical axis in the focal plane of the lens.

If the lens is more than one focal length from the grating, then somewhere beyond the Fourier transform plane, a real object will be formed (see Figure 3 for a schematic of the geometry of this situation). The interesting point of this arrangement is that the image that is formed can be understood as a light distribution that arises from the interference of the spatial frequency components of the diffracting object. In other words, the image is a Fourier synthesis of the spatial frequencies of the grating.

Thus, this imaging procedure can be thought of as a two-step process: first, a Fourier analysis is performed in which the frequency components are split up at the Fourier transform plane (which is

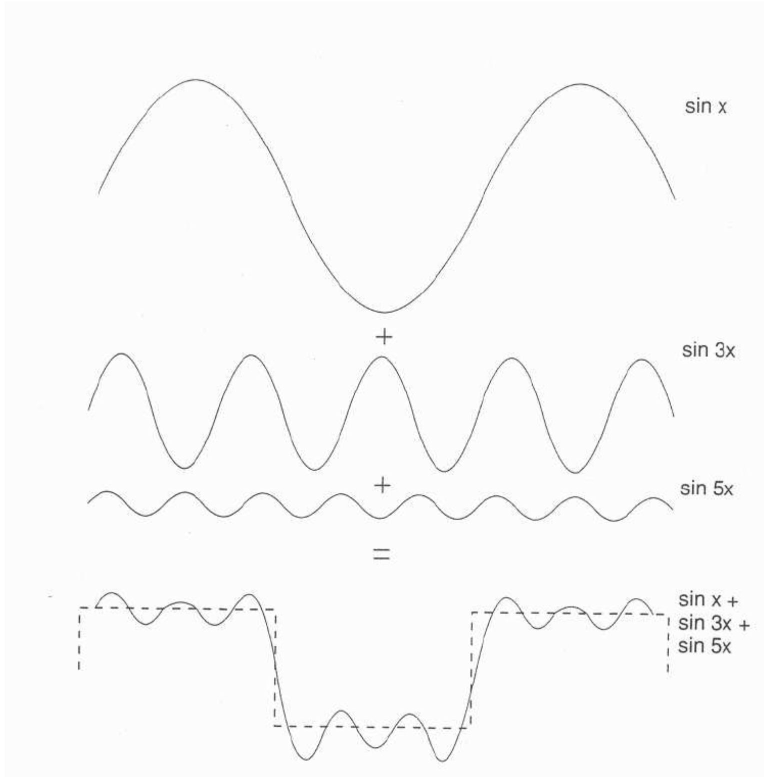

Figure 2: *Fourier synthesis of a square wave function.*

the focal plane of the lens); second, a Fourier synthesis is performed to create the image from these frequency components (which appears at the image plane). This approach to analyzing images was first proposed by Ernst Abbe.

One can see that the larger the lens used, the more of the widely diffracted (i.e., high frequency) components will be retained and focused to make the image. Although very little light is collected at the edge of the lens, it is this light that contributes to the fine detail in high-resolution images.

Since the light in the Fourier transform plane (Figure 3) is arranged according to increasing spatial frequency with radius, any intervention in that plane in the form of a mask which selectively obstructs and transmits the light will change the distribution of spatial frequencies present. This will in turn change the content of the image in a predictable way. This method of modifying an image by changing the spatial frequencies that are present is called spatial filtering.

There are a number of applications based on this approach to imaging. One of these "cleans up" a laser beam (see Figure 4). The irradiance distribution of the beam in many lasers is Gaussian as it exits the output mirror of the laser. However, dust and small imperfections in the lenses, windows, and surfaces it traverses or from which it reflects can produce irregularities in the irradiance pattern. The Gaussian distribution represents a low-frequency spatial variation in the beam, whereas the irregularities contain higher spatial frequencies. When the beam is focused with a lens, these variations are arranged according to their spatial frequencies. If a small pinhole whose diameter is sufficiently large to pass the low-frequency portion of the beam and block the high frequency part is placed at the focal plane (Fourier transform plane), the irregularities will be removed from the emerging beam and a "clean" beam will result.

Experiment

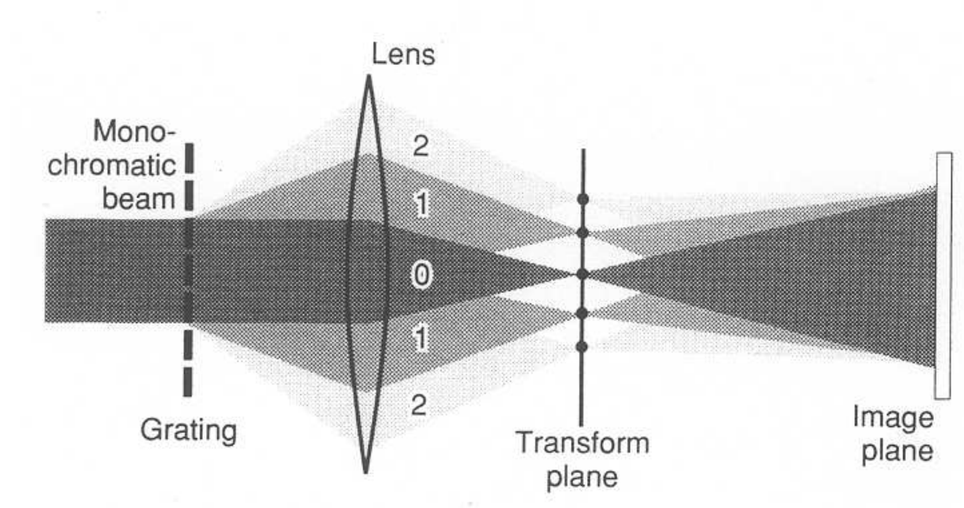

Figure 3: *Abbe theory of imaging.*

In this lab the frequency distribution of an object illuminated with a laser beam will be examined with a single lens placed after after a picture slide. The light distribution formed at the focal plane tells us the frequency content of the image, and by manipulating the light in that plane we can control the quality and content of the image to be displayed.

The laser beam has a smooth profile (i.e., it does not have any high-frequency components), and when it is focused it produces a single small spot. If we pass this beam through a grating or screen, then in the focal plane of the lens we will see several spots indicating that additional spatial frequency components have been added. We will build an experimental set up which will allow us to examine this feature. The setup is shown in Figure 5.

1. Mount a laser assembly (item A) on the far side of the optical table, and adjust the position such that the laser beam is parallel to the edge. Tape an index card with a small (2 mm) hole in it to the front of the laser so that the laser beam can pass through it. This card will be used to monitor the reflections from the optical components for alignment purposes as they are inserted into the beam.

   Warning: This experiment will be performed in a darkened room. Extreme care should be taken when using the laser. Your pupils will be expanded and will let in 60 times more light than a lighted room. Do not look at direct specular reflections or the direct laser beam. Please wear the appropriate eye protection when the laser is in use.

2. Mount a steering mirror assembly (item B) at the far corner of the table. Adjust the mirror so that the beam intersects the center of the mirror and reflects the beam at a right angle.

3. Place a second mirror steering assembly (item C) in line with the laser beam at the lower left corner of the table. Again adjust the mirror so that the reflected beam is at a right angle.

4. Set up a beam expander (item D) between the two mirror assemblies.

5. Mount the transform lens (focal length 150 mm) in a lens assembly (item E) at a distance 12 inches (305 mm) from the second steering mirror (item C). Center the lens in the laser beam path.

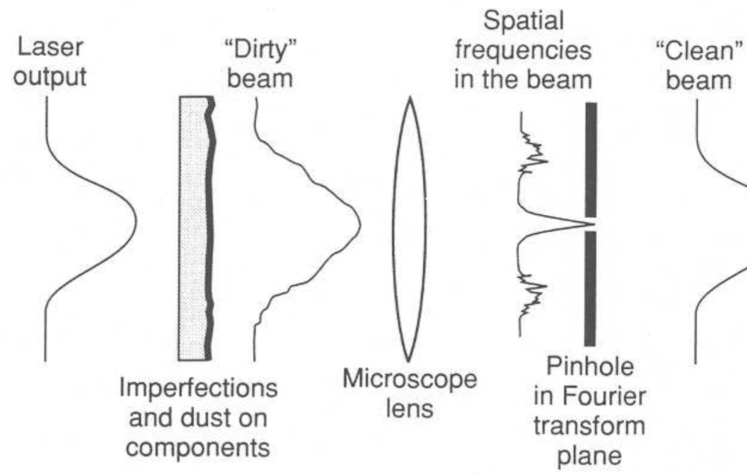

Figure 4: *Spatial filtering of a laser beam.*

6. Set up a target assembly and place a focus index card (item F) at the beam focus 150 mm behind the transform lens (item E). The plane of the card represents the focal plane of the lens.

7. Mount another target assembly (item G) at a distance 225 mm in front of the transform lens (item E). This slide holder will be used to hold the picture slides in the path of the laser beam.

8. The set up is now ready for the examination of slides. The image will be observed on an observation index card mounted about 450 mm behind the lens (300 mm behind the focus card) and will give an image about twice the size of the object slide image when the focus card is removed.

9. With no slide present, the laser beam produces a single focus spot. The position of the spot locates the "DC level" of illumination of the beam entering the lens. This corresponds to a spatial frequency of zero (or complete flatness). Any other spots appearing on the card indicate that other spatial frequencies are present. Remove the focus card and you will see uniform (DC level) illumination at the observation card.

10. Place the square mesh in the slide holder and replace the index card to the focal plane of the lens. On the focus card, you should see a square grid pattern of dots representing the the frequency content of the mesh in both the horizontal and vertical directions. Mark on the card with a pencil the location of these axes. The dots on the x and y axes represent the frequencies present in the horizontal and vertical directions respectively in the slide.

11. Remove the focus card and adjust the observation index card distance to achieve the sharpest image. This image can be manipulated by blocking certain frequencies with the focus card. To illustrate this, cut out a vertical slit in the focus card which will pass through only the vertical

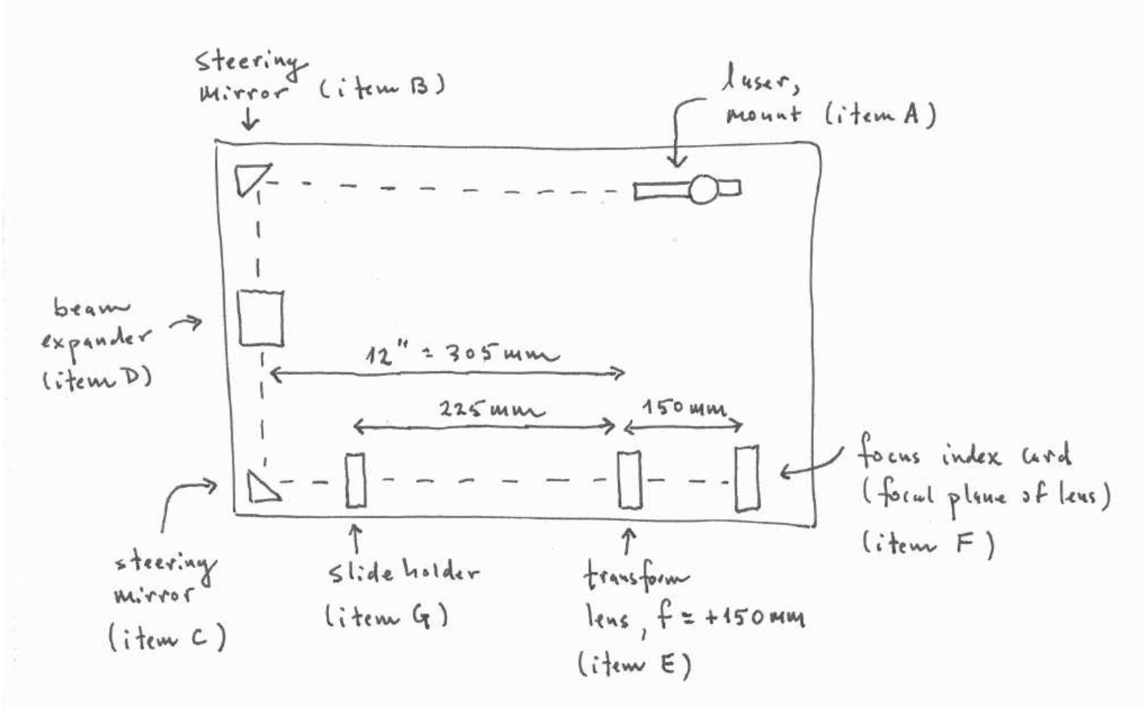

Figure 5: *Schematic of the experimental setup.*

frequency components (only the center horizontal component, the DC, gets through). Examine the image on the index card at the observation plane and record what you see.

12. Rotate the focus card so that the dots on the x-axis (horizontal direction) are passed. Record what you see.

13. Using another focus card, cut out a hole so that only the central focus spot (DC) is transmitted. You should see only uniform (flat) illumination on the observation card. This is the principle of the spatial filter and the "cleaning" of optical beams described above. The high spatial frequencies are removed by obstructing the unwanted harmonics, resulting in a low-pass filtered signal.

14. Make another filter in the form of a larger circular hole that passes only the central spot and the two adjacent spots. Record what you see on the observation card. Note that this removes sharp edges and produces a soft image (this is another low-pass filter at work). Alternately, make a special filter that will obstruct only the central spot by putting a drop of ink on a microscope slide and locating it at the center spot on the focal plane. Record your observations. This will remove the uniform illumination (DC) component and will leave the edges enhanced. This is a high-pass filter.

15. Replace the square mesh slide with a picture slide. Notice that this slide has lots of horizontal lines superimposed on it. This will be the original image which will be filtered. On the focus card the light distribution consists of an irregular distribution of light representing the many spatial frequencies present in the picture. Superimposed on this distribution is a set of faint spots aligned along the vertical axis and passing through the central spot. This line of spots represents the frequency content of the horizontal lines on the slide.

16. Attach with tape to the slide two pins (or toothpicks or thin objects) so that when it is placed back in the focal plane, the objects will obstruct the vertical line of spots. The central spot should

not be obscured. Record your observations.

17. Other cutouts may be used at the focal plane. For example, cut a hole in the index card so it obstructs the outside spots. These spots contain the high frequency information. Record your observations. Obstructing them results is a fuzzier picture at the observation plane.

Turn In

Turn in descriptions, recordings, and sketches of what you saw in each part of the experiment. Describe the effects seen in each case and explain their causes.